Explicit Link Between Periodic Covariance Functions and State Space Models

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INTRODUCTION

- Gaussian processes (GPs) [2] are commonly used modeling tools in non-parametric machine learning.
- Prior assumptions are encoded into the covariance function (kernel).
- We show that periodic covariance functions in GP regression can be rewritten as state space models.
- The model is written in terms of a series of stochastic resonators.
- Generalizes to quasi-periodic (almost periodic) covariance functions.

GAUSSIAN PROCESS REGRESSION

- Kernel representation: In GP regression the model functions f are assumed to be realizations from a GP prior, and the observations y_n, n = 1, 2, ..., n, corrupted by Gaussian noise: y_n = f(x_n) + e_n, where (e_n, x_n) ~ N(0, σ^2). Certain classes of covariance functions allow to work with the mathematical dual, where the Gaussian process is constructed as a solution to a m th order linear stochastic differential equation (SDE).
- State space representation: The GP regression problem can also be written as:

\[
\begin{align*}
\dot{f}(t) = Fr(t) + Lw(t) \\
y_n = Hf(x_n) + r_n
\end{align*}
\]

where \(w(t)\) is a multi-dimensional white noise process with spectral density \(Q_0\). The model is defined by the feedback matrix \(F\), the noise effect matrix \(L\), the spectral density \(Q_0\), the stationary covariance \(P_0\), and the observation model \(H\).
- The inference problem can now be solved using Kalman filtering [3] in O(n) time complexity.

CO2 PRODUCTION IN FINLAND


CONCLUSIONS

- We have established the explicit connection between periodic covariance functions and state space models.
- This link enables the use of efficient sequential inference methods to solve periodic GP regression problems in \(O(n)\) time complexity.
- The approximation converges uniformly and a rough upper bound for the error can be given in closed-form.
- This is a 'best of both worlds' approach; it brings together the conventional model specification and hyperparameterization of GPs with the computational efficiency of state space models.

EXAMPLE IMPLEMENTATION

- An example implementation is available on the author’s web page:
  http://becs.aalto.fi/~asolin/
- The method is also a part of the GPstuff toolbox for Matlab/Octave.

REFERENCES


DEMONSTRATIONS

- A simulated example showing the computational efficiency.
- Prediction of CO2 levels using weekly data (see [2]), where we compare the approximation to the full GP result.
- Explaining the periodic variation in the number of births per day in the US (see [4]).